Maximum Throughput in the H-filter

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An interesting question is what is the maximum throughput you can get through an H-filter assuming that any device is possible to construct and considering all particles to be point-like.

First, I assume that the device works only up to a certain Reynolds number, f, beyond which the laminar flow is destroyed and convective mixing occurs. The Reynolds number is

$$R_e = \frac{\rho v h}{\eta}. \tag{1}$$

Second the volume flow rate, Q is given by

$$Q = vhw = \frac{\eta w R_e}{\rho}. \tag{2}$$

So that the maximum throughput, Q_{max} is given by

$$Q_{max} = f\frac{\eta w}{\rho}. \tag{3}$$

Now the particle, with diffusion coefficient D, must spend enough time $t = l/v$ in the channel to diffuse across a distance h (which takes $\tau = l^2/D$), so

$$\frac{vh^2}{l} \leq D. \tag{4}$$

At a Reynolds number of f, this equation can be written as

$$\frac{h}{l} \leq \frac{D\rho}{f\eta}. \tag{5}$$

For the maximum throughput (equation 3), we want f and w as large as possible, assuming the fluid (ρ and η) is fixed. The width, w, can be increased without limit. But the maximum Reynolds number, f, is limited to something less than 2000. The exact value depends on the geometry and various things.

Since equation 5 also puts a constraint on h/l we can say that the *maximum throughput per unit width is obtained when*

$$\frac{l}{h} = \frac{f\eta}{\rho D}. \tag{6}$$

In other words the aspect ratio (a *different* aspect ratio than we have been talking about and Giddings claims everything greater than 50) should be as large as possible. But, any larger than $f\eta/\rho D$ is a waste, becuase the flow rate would have to be slowed down. For a typical protein ($D = 10^{-6}$cm^2/s) in water, this aspect ratio would ideally be about $f \times 10^4$.